> Suppose one out of every million people is a terrorist (if anything, an overestimate), and you've got a machine that can determine whether someone is a terrorist with 99.9 percent accuracy. You've used the machine on Mr. X, and it gives a positive result. What are the odds that Mr. X is a terrorist? Here's the answer: a 0.1 percent chance — which is to say, the 99.9 percent accurate test will give you the wrong answer 99.9 percent of the time.[1]

Let $X = 1$ be the event that Mr. X is a terrorist with probability one in a million.

$$P(X{=}1) = 0.000001$$

By complementarity

$$P(X{=}0) = 0.999999$$

Let $M = 1$ be the event that the machine gives a positive result. Since the machine is 99.9 percent accurate when $X = 1$ we have

$$P(M{=}1 \mid X{=}1) = 0.999$$

By complementarity

$$P(M{=}0 \mid X{=}1) = 0.001$$

The probability that the machine produces an incorrect result is

$$\begin{aligned} P(M{\neq}X) &= P(M{=}0 \cap X{=}1) + P(M{=}1 \cap X{=}0) \\ &= P(M{=}0 \mid X{=}1)P(X{=}1) + P(M{=}1 \mid X{=}0)P(X{=}0) \end{aligned}$$

Rearranging

$$P(M{=}1 \mid X{=}0) = \frac{P(M{\neq}X) - P(M{=}0 \mid X{=}1)P(X{=}1)}{P(X{=}0)}$$

Solve for the false positive probability $P(M{=}1 \mid X{=}0)$.

$$P(M{=}1 \mid X{=}0) = \frac{0.001 - 0.001P(X{=}1)}{P(X{=}0)} = 0.001$$

What we want to determine is $P(X{=}1 \mid M{=}1)$. That is, the probability that $X = 1$ when the machine gives a positive result. From the definition of conditional probability we have

$$P(X{=}1 \mid M{=}1) = \frac{P(X{=}1 \cap M{=}1)}{P(M{=}1)}$$

Also by definition

$$P(M{=}1 \mid X{=}1) = \frac{P(M{=}1 \cap X{=}1)}{P(X{=}1)} = \frac{P(X{=}1 \cap M{=}1)}{P(X{=}1)}$$

It follows that

$$P(X{=}1 \cap M{=}1) = P(M{=}1 \mid X{=}1)P(X{=}1)$$

To determine $P(M{=}1)$ we use total probability.

$$P(M{=}1) = P(M{=}1 \mid X{=}1)P(X{=}1) + P(M{=}1 \mid X{=}0)P(X{=}0)$$

The following Eigenmath code computes $P(X{=}1 \mid M{=}1)$ which is the probability that the machine is correct when it gives a positive result.

[1] Cooper, Ryan. *The simple math problem that blows apart the NSA's surveillance justifications.* http://theweek.com/articles/547119/simple-math-problem-that-blows-apart-nsas-surveillance-justifications

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PX1 = 1/1000000                      -- probability that X=1
PX0 = 1 - PX1                        -- probability that X=0
PM1X1 = 999/1000                     -- probability that machine is correct
PM1X0 = 1/1000                       -- probability of false positive
PM1 = PM1X1 * PX1 + PM1X0 * PX0      -- probability that M=1
PM1X1 * PX1 / PM1                    -- probability that X=1 when M=1
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$\frac{1}{1002}$

So the probability is indeed about 0.1 percent.